Engineering Experiment Station

NEW MEXICO STATE UNIVERSITY

ADVANCED STUDY OF VIDEO SIGNAL PROCESSING IN LOW SIGNAL TO NOISE ENVIRONMENTS

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Frank Carden
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A Semi-Annual Progress Report

Submitted to

NATIONAL AERONAUTICAL SPACE ADMINISTRATION
WASHINGTON, D. C.

NASA RESEARCH GRANT NGR-32-003-037

Electrical Engineering Department
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June 1972-November 1972

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I. INTERPOLATIVE ESTIMATION OF A VIDEO PROCESS USING INTER-FRAME CORRELATION PROPERTIES

ABSTRACT

The frame to frame correlation properties of the video process are utilized to reduce the mean squared error of the demodulated video where zero mean noise is a factor. An interpolative estimator is used for continuous estimation with the output process delayed in time by one frame. Theoretical development shows that for the model herein developed reduction of the mean squared error by 1.0 to 4.0 dB is possible for parameter ranges of interest.

II. AN APPLICATION OF INTERPOLATIVE ESTIMATION TO THE APOLLO 17 TELEVISION PROBLEM

ABSTRACT

The theory developed in previously submitted work on interpolative estimation using inter-frame correlation properties of a video process is applied to the Apollo 17 parameters to yield a model for application on that mission.

I. INTERPOLATIVE ESTIMATION OF A VIDEO PROCESS USING INTER-FRAME CORRELATION PROPERTIES

INTRODUCTION

In noisy environments television signals become corrupted and result in image displays which are deteriorated from the original image. Most images can be considered to have a scanned output which is positive definite, and which then has some non-zero mean m. The assumption that the noise has zero mean is generally valid. For this class of images some form of a processor is desired which utilizes the inter-frame correlation properties of television signals to reduce the mean squared error of the recovered image.

Such correlation based estimation of a video signal requires large amounts of storage, a drawback which has eliminated frame to frame correlation estimation for most numerical methods. The following development, using only frame delays, makes possible application of disk recorders and delay lines for implementation of the solution offered.

The correlation between frames in a video process is generally obvious, with a small percentage of picture elements (pixels) changing from frame to frame [1]. The model herein developed uses the information contained in the current received pixel along with the corresponding pixel in the previous and succeeding frames to arrive at a minimum mean squared error estimate of the current value.

Development is restricted to a three frame system, but the extension to more frames is obvious and straightforward.

DEVELOPMENT OF THE MODEL

Let $x(t)$ be a weak sense stationary random process,

$$x(t) = c(t) + n(t) + m,$$

$E[c(t)] = E[n(t)] = 0,$

$E[c^2(t)] = \sigma_c^2,$

$E[n^2(t)] = \sigma_n^2,$ and

$E[x(t)] = m.$

For $t \varepsilon [0,T]$ where T is the frame time, let

$$x(t + nT) = x_n(t)$$

where

$$x_n(t) = c_n(t) + n_n(t) + m.$$

Letting the argument be understood and dropping it,

$x_n = c_n + n_n + m$, where x_n belongs to the n^{th} frame.

Making the assumptions

$E[n_i\ n_j] = \sigma_n^2\ \delta_{ij}$ where δ_{ij} is the Kronecker delta function [2],

$E[c_i\ c_j] = \rho_{(i-j)}\ \sigma_c^2 = \rho_{(j-i)}\ \sigma_c^2$, and

$E[c_i\ n_j] = 0,$

then

$$E[x_i\, x_j] = E[(c_i + n_i + m)(c_j + n_j + m)]$$

$$= \rho_{(i-j)}\, \sigma_c^2 + \sigma_n^2\, \delta_{ij} + m^2$$

and

$$E[c_i\, x_j] = \rho_{(i-j)}\, \sigma_c^2 \qquad \text{for all } i, j.$$

Now then let c_n^* be the linear minimum mean squared error estimate of c_n using the data set $\{x_{n-1}, x_n, x_{n+1}\}$, where c_n^* is an unbiased estimator of $E[c_n]$.

$$c_n^* = a_{n-1}(x_{n-1} - m) + a_n(x_n - m) + a_{n+1}(x_{n+1} - m)$$

$$E[c_n^*] = a_{n-1}E[(x_{n-1} - m)] + a_nE[(x_n - m)] + a_{n+1}E[(x_{n+1} - m)]$$

$$= a_{n-1}E[c_{n-1} + n_{n-1}] + a_nE[c_n + n_n] + a_{n+1}E[c_{n+1} + n_{n+1}]$$

$= 0$ therefore unbiased estimator.

By the orthogonality principle [3]

$$E[(c_n - c_n^*)\, x_i] = 0 \qquad i \in \{n-1, n, n+1\}$$

therefore $E[c_n\, x_i] = E[c_n^*\, x_i]$

but $E[c_n\, x_i] = \rho_{(n-i)}\, \sigma_c^2$ so

$$\rho_{(n-i)}\, \sigma_c^2 = E[c_n^*\, x_i]$$

$$= a_{n-1}E[(c_{n-1} + n_{n-1})\, x_i] + a_nE[(c_n + n_n)\, x_i]$$

$$+ a_{n+1}E[(c_{n+1} + n_{n+1})\, x_i]$$

$$= a_{n-1}(\rho_{(n-1-i)}\ \sigma_c^2 + \sigma_n^2\ \delta_{n-1,i}) + a_n(\rho_{(n-i)}\ \sigma_c^2 + \sigma_n^2\ \delta_{ni})$$

$$+ a_{n+1}(\rho_{(n+1-i)}\ \sigma_c^2 + \sigma_n^2\ \delta_{n+1,i})$$

letting i = n-1, n, n + 1 successively

$$\rho_1\ \sigma_c^2 = a_{n-1}(\sigma_c^2 + \sigma_n^2) + a_n(\rho_1\ \sigma_c^2) + a_{n+1}(\rho_2\ \sigma_c^2)$$

$$\sigma_c^2 = a_{n-1}(\rho_1\ \sigma_c^2) + a_n(\sigma_c^2 + \sigma_n^2) + a_{n+1}(\rho_1\ \sigma_c^2)$$

$$\rho_1\ \sigma_c^2 = a_{n-1}(\rho_2\ \sigma_c^2) + a_n(\rho_1\ \sigma_c^2) + a_{n+1}(\sigma_c^2 + \sigma_n^2)$$

It can be seen from the symmetry of the matrix of coefficients that application of Cramer's rule will yield $a_{n-1} = a_{n+1}$ so simplify to

$$\rho_1\ \sigma_c^2 = a_{n-1}[\sigma_c^2(1 + \rho_2) + \sigma_n^2] + a_n(\rho_1\ \sigma_c^2)$$

$$\sigma_c^2 = a_{n-1}(2\rho_1\ \sigma_c^2) + a_n(\sigma_c^2 + \sigma_n^2)$$

The system determinant Δ follows where γ is $\frac{\sigma_c^2}{\sigma_n^2}$

$$\Delta = (1 + \rho_2 + \frac{1}{\gamma})(1 + \frac{1}{\gamma}) - 2\rho_1^2$$

and the coefficients

$$a_n = \frac{1}{\Delta}(1 + \rho_2 + \frac{1}{\gamma} - 2\rho_1^2)$$

$$a_{n-1} = \frac{1}{\Delta}\left(\frac{\rho_1}{\gamma}\right).$$

The processor is shown in Figure 1.

NOISE REDUCTION PROPERTIES

Since the input "signal" to the processor is c(t) and the input

noise is n(t) then the input mean squared error is

$$MSE_1 = E[(x(t) - m - c(t))^2]$$

$$= E[(c(t) + n(t) - c(t))^2]$$

$$= E[n^2(t)] = \sigma_n^2 .$$

So that the input mean squared error is just the variance of the noise.

Now then the output mean squared error is MSE_2 where

$$MSE_2 = E[(c(t) - c^*(t))^2]$$

$$= E[c_n^{*2}] + E[c_n^2] - 2E[c_n\ c_n^*]$$

but

$$E[c_n^{*2}] = E\{[a_{n-1}(c_{n-1} + n_{n-1}) + a_n(c_n + n_n) + a_{n+1}(c_{n+1} + n_{n+1})]^2\}$$

$$= a_{n-1}^2\ E[(c_{n-1} + n_{n-1})^2] + a_n^2\ E[(c_n + n_n)^2]$$

$$+ a_{n+1}^2\ E[(c_{n+1} + n_{n+1})^2] + 2a_n\ a_{n+1}\ E[(c_n + n_n)(c_{n+1} + n_{n+1})]$$

$$+ 2a_{n-1}\ a_n\ E[(c_n + n_n)(c_{n-1} + n_{n-1})]$$

$$+ 2a_{n+1}\ a_{n-1}\ E[(c_{n+1} + n_{n+1})(c_{n-1} + n_{n-1})]$$

$$= (2a_{n-1}^2 + a_n^2)(\sigma_c^2 + \sigma_n^2) + 4a_n\ a_{n-1}\ \rho_1\ \sigma_c^2 + 2a_{n-1}^2\ \rho_2\ \sigma_c^2 .$$

Now

$$E[c_n\ c_n^*] = E[a_{n-1}(c_{n-1} + n_{n-1})c_n + a_n(c_n + n_n)c_n + a_{n+1}(c_{n+1} + n_{n+1})c_n]$$

$$= 2a_{n-1}\ \rho_1\ \sigma_c^2 + a_n\ \sigma_c^2,$$

so

$$MSE_2 = (2a_{n-1}^2 + a_n^2)(\sigma_c^2 + \sigma_n^2) + 4a_n\, a_{n-1}\, \rho_1\, \sigma_c^2 + 2a_{n-1}^2\, \rho_2\, \sigma_c^2$$

$$- 2\sigma_c^2(2a_{n-1}\, \rho_1 + a_n) + \sigma_c^2 .$$

The improvement in the mean squared error then is

$$IMP = -10 \log \frac{MSE_2}{MSE_1} ,$$

and for some typical values of parameters under the assumption; e.g.

$\rho_1 = K$

$\rho_j = K^j$ [4]

$\sigma_c^2 = 1/12$ which corresponds to c(t) distributed uniform on $[-\frac{1}{2}, \frac{1}{2}]$,

the improvement is given in Figure 2.

SENSITIVITY CONSIDERATIONS

Under conditions of variation in operating parameters the sensitivity of the model with fixed coefficients to these changes is of interest.

It is to be noted that the input SNR is computed only on the varying part of the "signal", i.e. c(t), so that the power in the D.C. component is ignored.

Choosing an input SNR of 8.41 dB and a K of .90 the sensitivity of the model to changes about this point is shown in Figure 3. It can be concluded from these results that small variations in γ and K about a fixed value will not appreciably effect performance.

AN APPLICATION

The theory herein derived was applied at the request of NASA to the

Apollo 17 television problem. Since the Apollo television system operates in two different modes, results were obtained for the furnished parameters of SNR = 9 dB and SNR = 17 dB. In Figure 4 the required coefficients for the processor are graphically shown for the first case over a large variation in K. Figure 5 displays the same information for the second case. Since K may vary widely due to differing video material, it should be determined by the user. Figure 6 shows the improvement obtainable for each of the modes under consideration.

VERIFICATION OF THE VALIDITY OF THE MODEL

In an attempt to verify the validity of the model three conditions may be examined, i.e.

1. That a_n goes to 1.0 as γ goes to infinity, and that a_{n-1} goes to 0.
2. That a_n goes to a_{n-1} as K goes to 1.
3. That a_{n-1} goes to zero as K goes to zero.

Condition 1 can be seen to be satisfied by comparing Figures 4 and 5, as well as examining the defining equations. The second condition is demonstrated in Figure 4 but may be easily shown from the equation. The third condition is obvious from the defining equation for a_{n-1}. Hence the equations defining the model comply with three important boundary conditions increasing confidence in their validity.

SUMMARY

A model has been developed and analyzed numerically yielding a theoretical reduction in mean squared error of a video process based upon the inter-frame correlation properties of the process. The theoretical

improvement obtained is substantial, enough to justify experimental verification. Analytical verification was offered by analysis of boundary conditions. Implimentation of the model is straightforward, and a completed processor should be realizable at reasonable cost.

REFERENCES

[1] Haskell, B. G., et al., "Interframe Coding of Videotelephone Pictures," IEEE Proceedings, Vol. 60, No. 7, July 1972.

[2] Butkov, Eugene, Mathematical Physics, pp. 430, Addison-Wesley, 1968.

[3] Papoulis, A., Probability, Random Variables and Stochastic Processes, McGraw-Hill, 1965.

[4] Franks, L. E., "A Model for the Random Video Process," B.S.T.J., Vol. 45, No. 4, April 1966.

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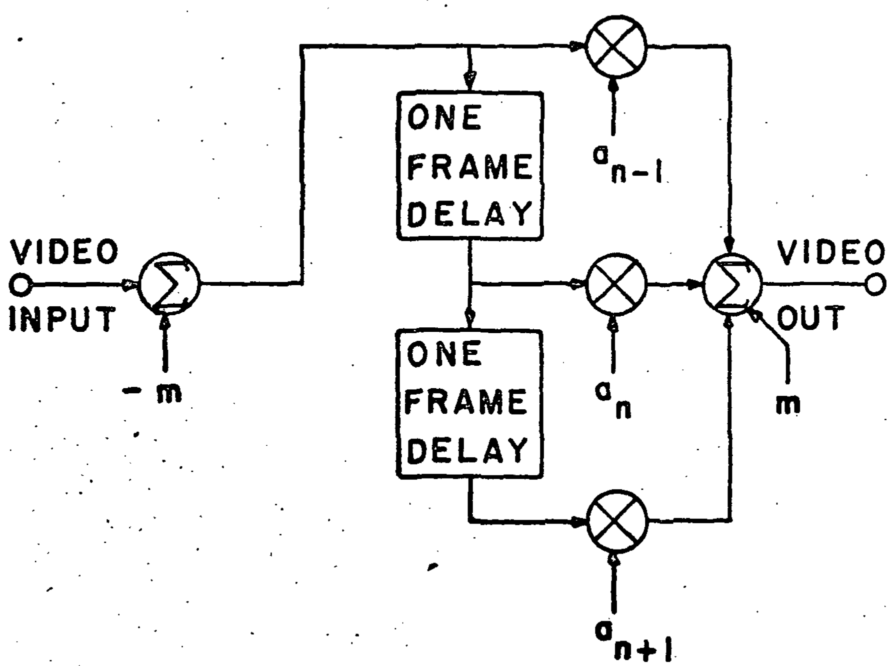
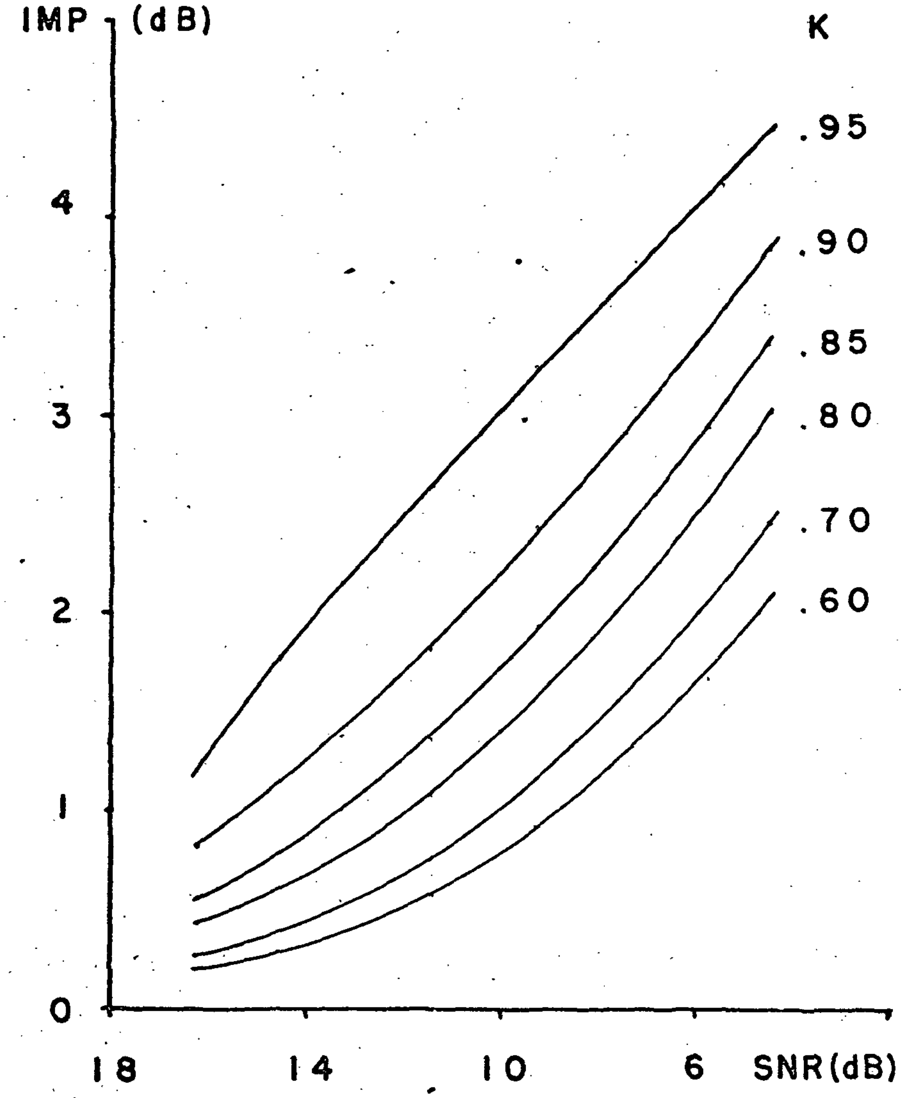


LINEAR MINIMUM MSE ESTIMATOR

FIGURE 1



MSE IMPROVEMENT

FIGURE 2

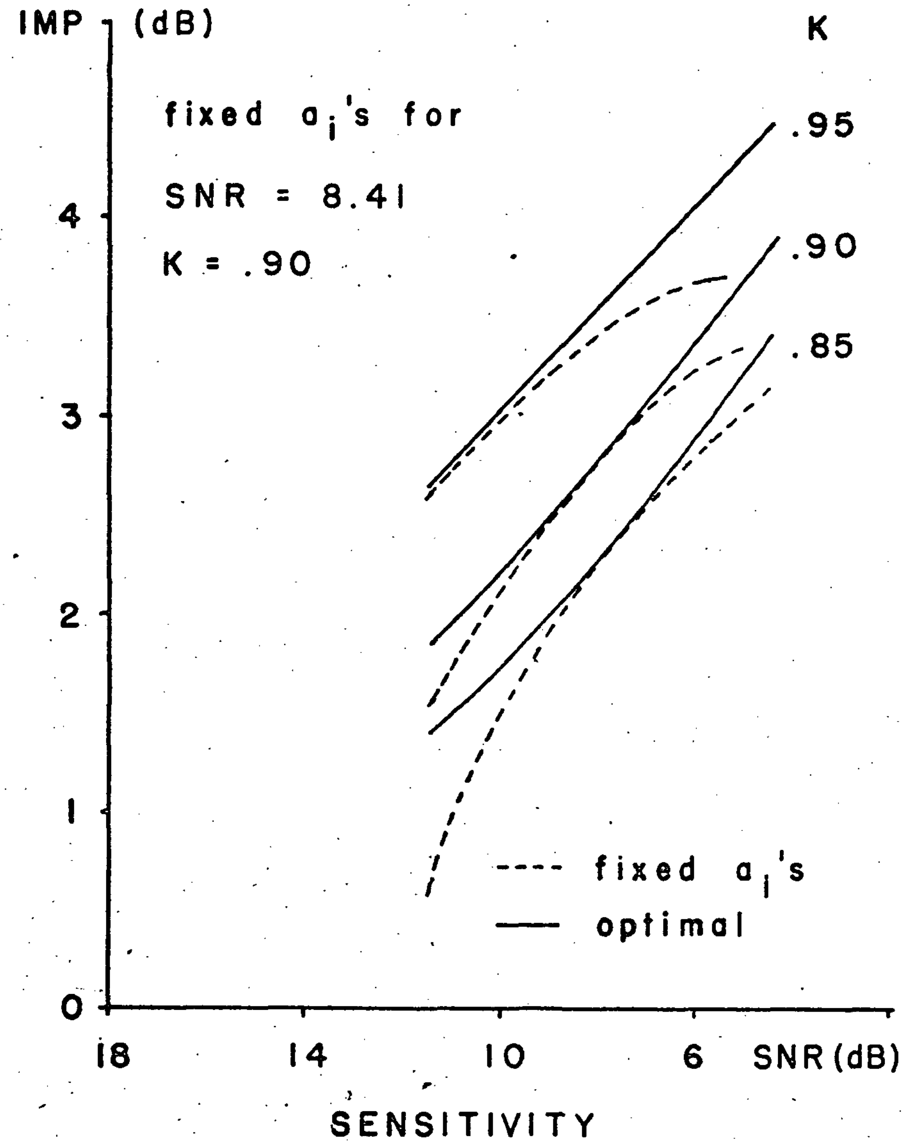
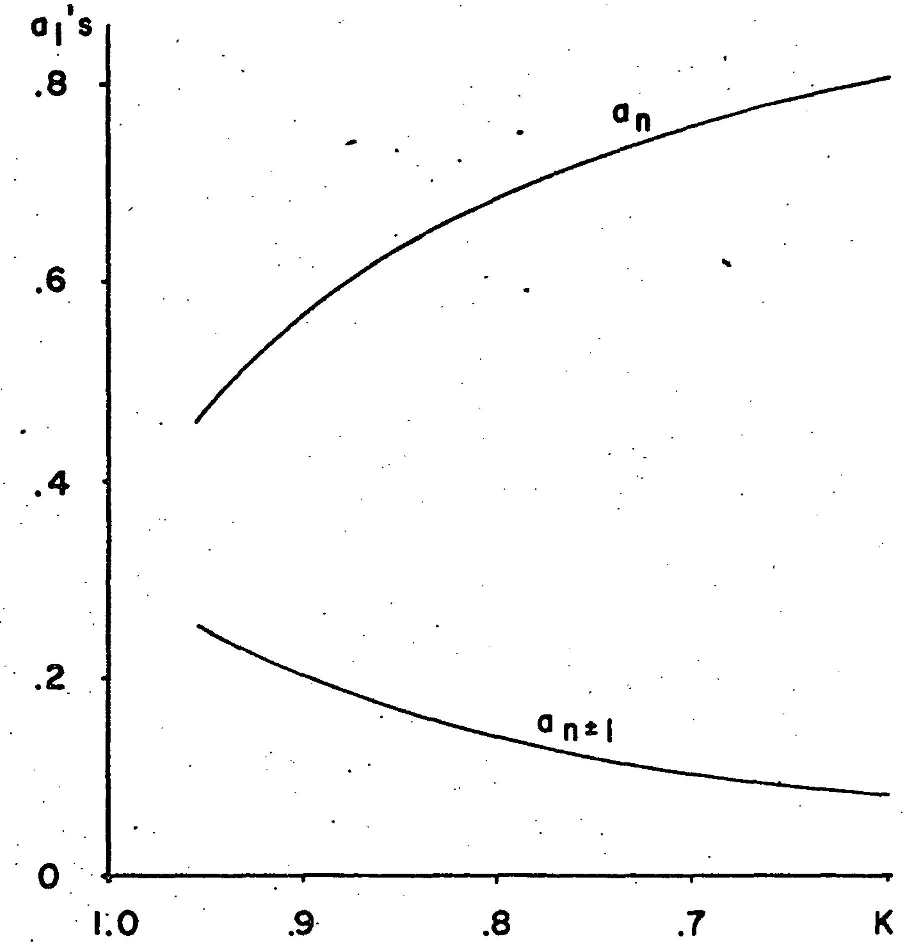
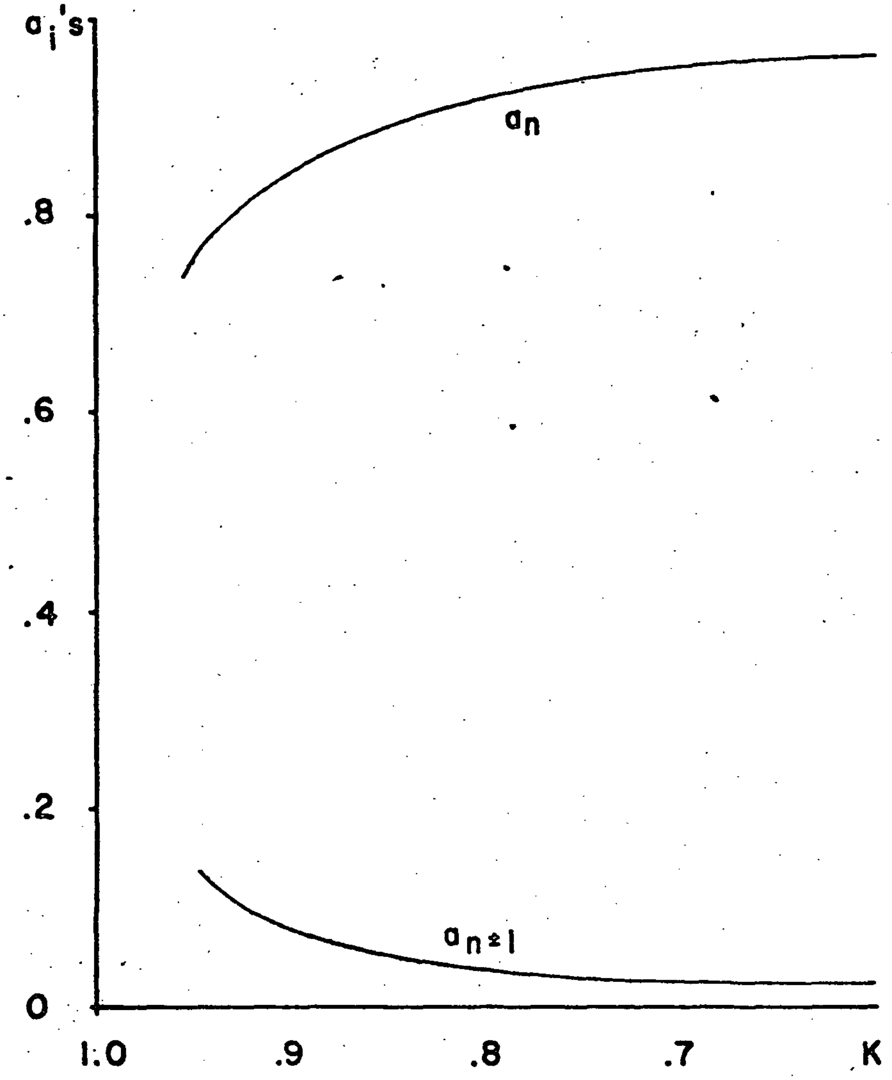

IMP (dB)
K
fixed a_i's for
SNR = 8.41
K = .90
.95
.90
.85
4
3
2
1
0
18
14
10
6
SNR (dB)
---- fixed a_i's
—— optimal
SENSITIVITY

FIGURE 3



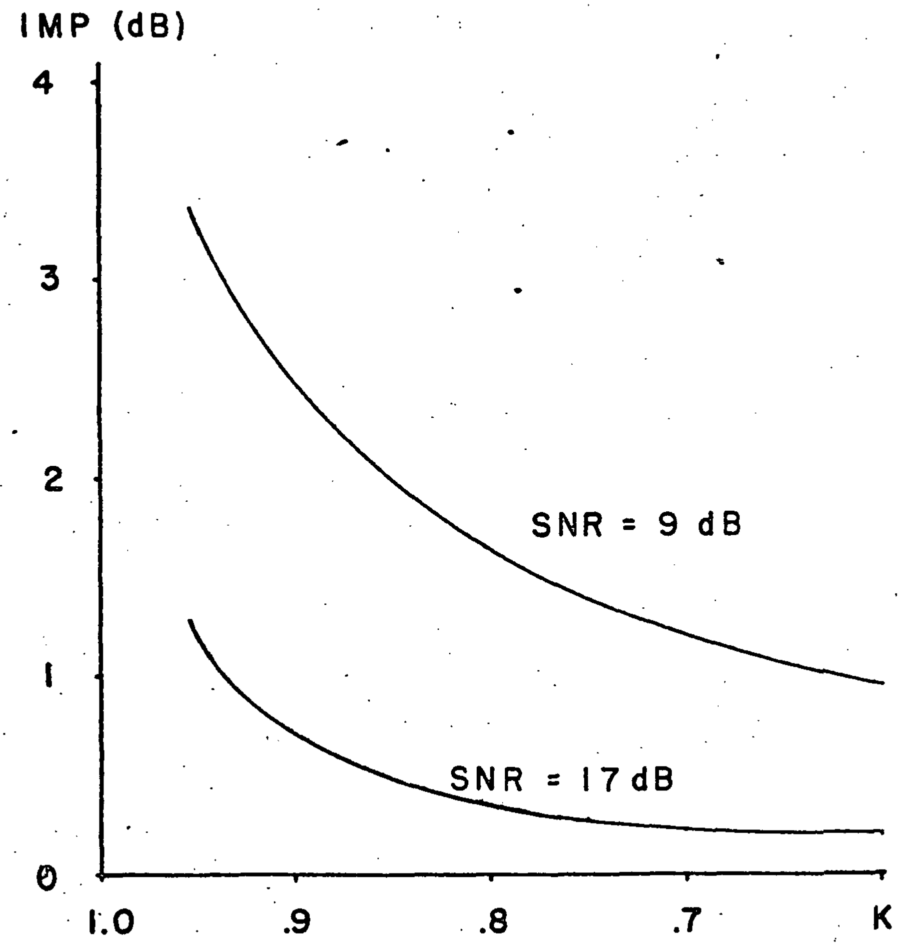
COEFFICIENTS FOR SNR OF 9 dB

FIGURE 4



COEFFICIENTS FOR SNR OF 17 dB

FIGURE 5



IMPROVEMENT FOR SNR OF 9 AND 17 dB

FIGURE 6

II. AN APPLICATION OF INTERPOLATIVE ESTIMATION TO THE APOLLO 17 TELEVISION PROBLEM

INTRODUCTION

Previous work at New Mexico State University indicated substantial reductions were possible in the mean squared error of a video process by taking full advantage of the inter-frame correlation properties and developing a linear minimum mean squared error interpolative estimator to yield video delayed by one frame. The proposed model is shown in Figure 1, with the derived improvements over a wide range of input signal to noise ratios and correlation coefficients shown in Figure 2. Sensitivity to small variations in SNR and K with a fixed coefficient model was determined to be within usable limits, with these results shown in Figure 3.

APPLICATION TO APOLLO 17

Parameters furnished by NASA were a signal to noise ratio for two separate modes of operation, one of 9 dB and one of 17 dB. The following assumptions were made about this data:

(1) the above modes could be anticipated and adjustments made in the processor,

(2) that these figures represented the SNR of the demodulated video.

Assumption 2 is the difficult one, since the television is FM modulated onto a carrier, and FM normally is associated with SNR improvements. Also the demodulated video is low pass filtered. The assumption should hold well, however, if the video is narrow-band FM modulated onto the carrier, and the SNRs given correspond to the two-sided SNR in the IF over the low-passed bandwidth of the video about the carrier.

An analysis of the data given then yields the required coefficients shown in Figure 4 for an SNR of 9 dB, and Figure 5 for an SNR of 17 dB. Improvements obtainable are shown in Figure 6. The coefficients and improvements for the single value K = .9 are:

SNR = 9 dB $\quad a_n = .565 \quad a_{n-1} = .202 \quad$ Imp = 2.48 dB.

SNR = 17 dB $\quad a_n = .852 \quad a_{n-1} = .073 \quad$ Imp = 0.69 dB.

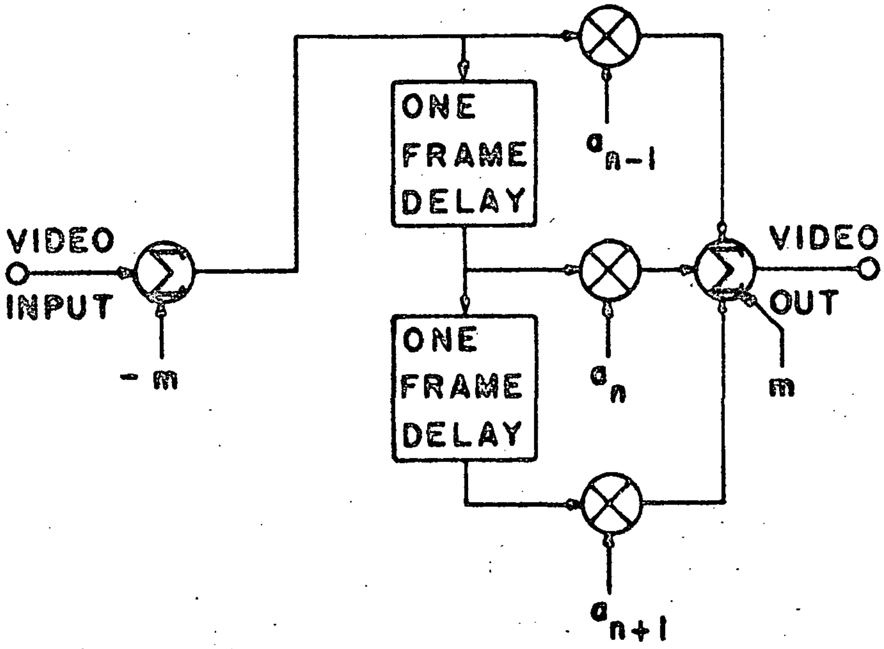
As K is determined other parameters may be found from the appropriate figures.

SUMMARY

A previously developed model was applied to the Apollo 17 television problem and parameters developed for application of the model. Improvements for each mode of operation are shown to be possible, with improvements in excess of 2 dB for some parameter values.

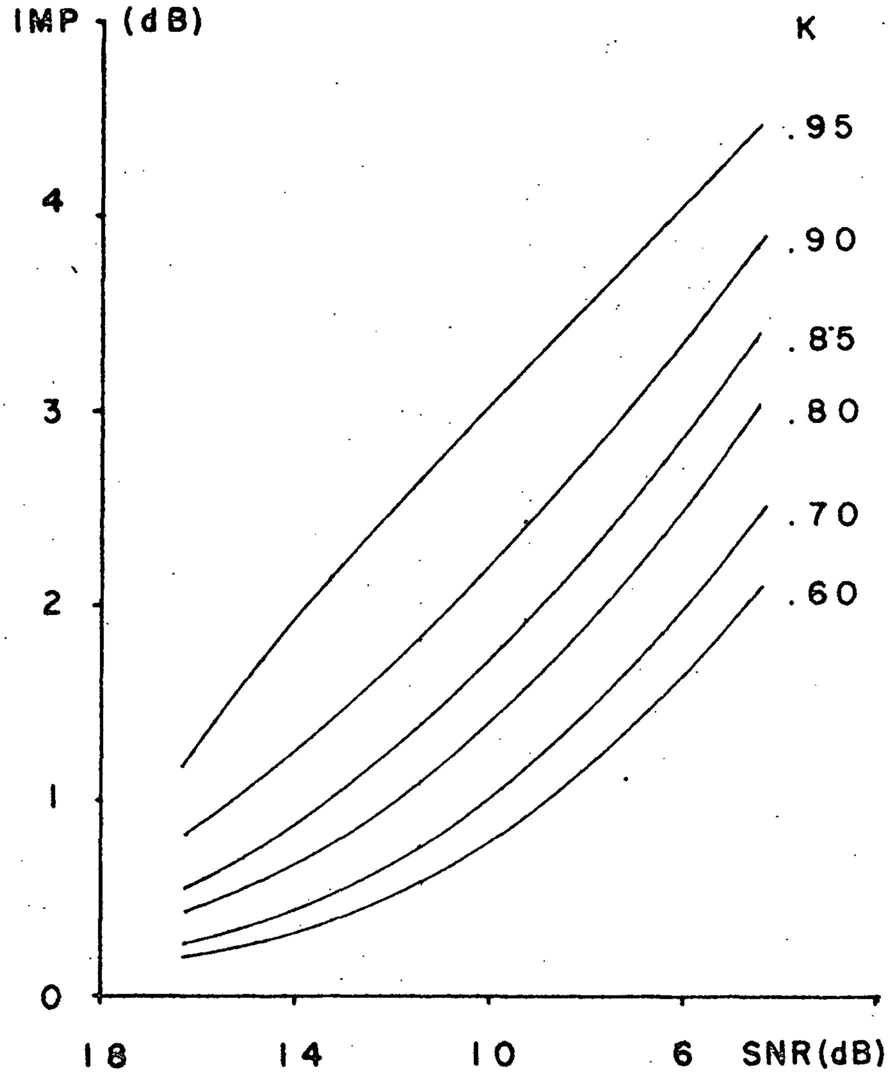
REFERENCE

A. L. Gilbert and F. F. Carden, "Interpolative Estimation of a Video Process using Inter-Frame Correlation Properties," to be published. Copy furnished herewith.



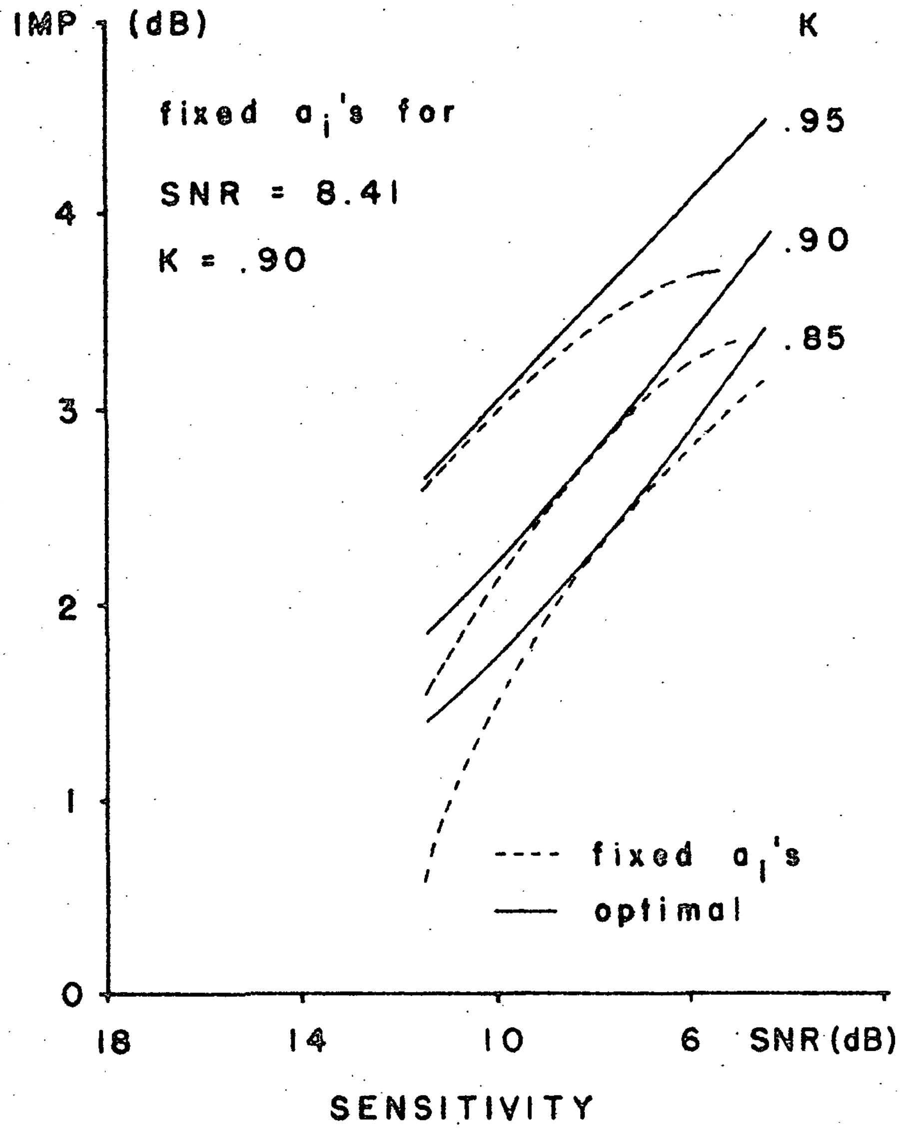
LINEAR MINIMUM MSE ESTIMATOR

FIGURE 1



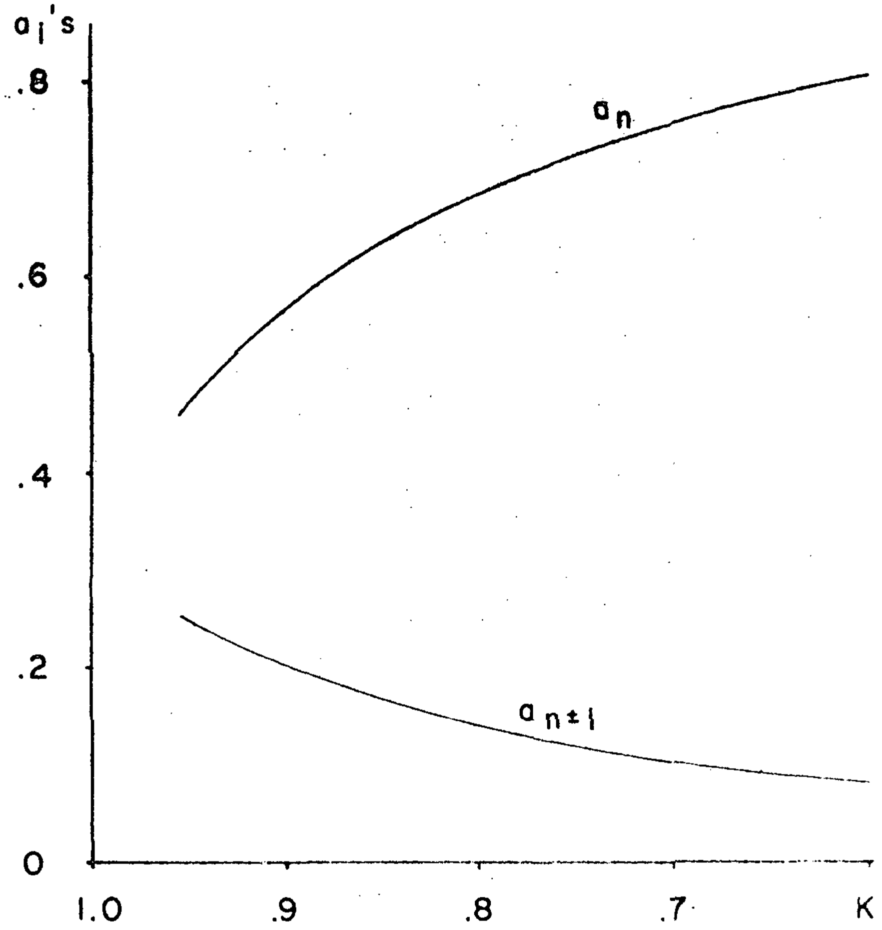
MSE IMPROVEMENT

FIGURE 2



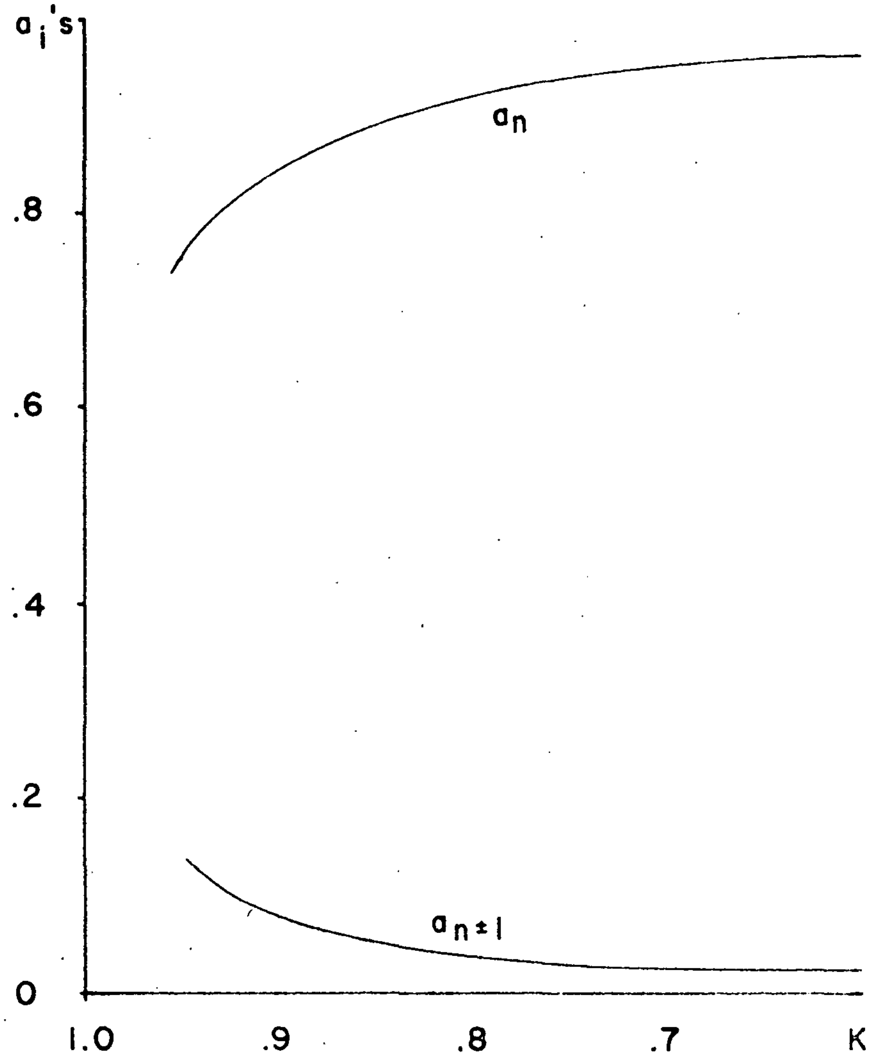
IMP (dB)
K
fixed a_i's for
SNR = 8.41
K = .90
.95
.90
.85
4
3
2
1
0
fixed a_i's
optimal
18
14
10
6
SNR (dB)
SENSITIVITY

FIGURE 3



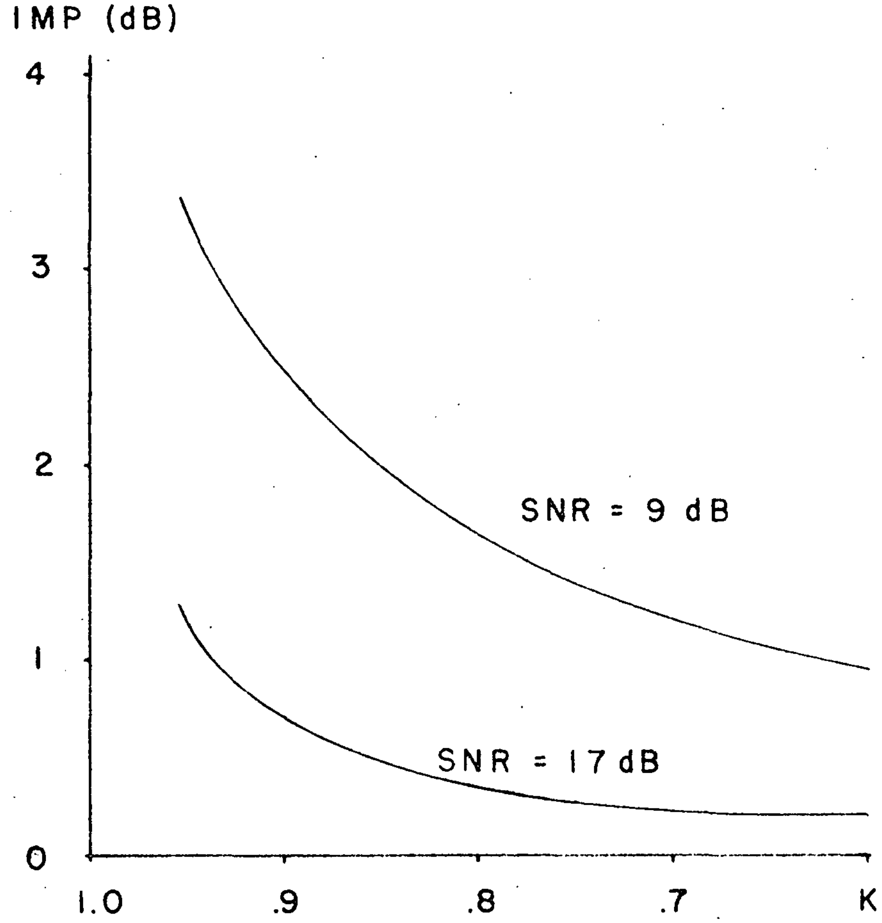
COEFFICIENTS FOR SNR OF 9 dB

FIGURE 4



COEFFICIENTS FOR SNR OF 17 dB

FIGURE 5



IMPROVEMENT FOR SNR OF 9 AND 17 dB

FIGURE 6

APPENDIX I

DISK RECORDER SPECIFICATIONS - Two alternative configurations.

Configuration 1

Number of video channels	4
Capability	Monochrome

Configuration 2

Number of video channels	8
Capability	Color

Common

Channel configuration	Each channel to have common REC/PB head with mode switching accomplished by TTL/DTL levels. Separate REC/PB amplifiers.
Channel dependency	Each channel independently available for RECORD or PLAYBACK mode.
Switching time REC/PB, PB/REC	20 nanosec maximum
Channel select imputs	One per channel, high level specifying one mode, low level the other.
Video channel input, output impedence	75 ohms each
Input video	composite or non-composite, 1V p-p.
Output video	composite or non-composite, 1V p-p.
Bandwidth	5.0 Mhz $\pm$ 3 dB.
Signal to noise ratio	40 dB or greater.
Channel capacity	one NTSC video frame
Servo synchronization	Sync track mode or external syncronization to video sync signal. Both required.
Synchronization generation	TTL/DTL compatible composite sync output from 15750 pps and 30 pps pulse train.

Time base stability	Peak to peak jitter not to exceed 50 nanosec.
Input voltage	115VAC 60Hz.
Mounting	Table-top portable.
Delivery	30 days ARO.